Invited Talks and Tutorials

A Constructive Version of the Lusin Separation Theorem

Peter Aczel

Departments of Mathematics and Computer Science
University of Manchester
Manchester, M13 9PL, United Kingdom
petera@cs.man.ac.uk

Abstract

The Lusin Separation Theorem is one of the fundamental early results of classical descriptive set theory. It states that if A_1, A_2 are disjoint analytic subsets of Baire space then they are Borel separable. Yiannis Moschovakis gives two proofs in his book, "Descriptive Set Theory". One proof is obviously highly non-constructive. The other would appear to be constructive and uses Bar Induction and Bar Recursion. But the exact constructive status of the result needs some care.

By constructively strengthening both the assumption and, in compensation, the conclusion of the implication we are able to give a proof in the formal system CZF of constructive set theory. We avoid using Bar Induction by using a strong inductive point-free notion of disjointness. We also strengthen the conclusion and avoid using any choice principles by using a point-free notion of Borel separation. So our main result would be acceptable in Bishop style constructive mathematics, even without countable choice. Moreover our main result, combined with Bar Induction, gives a pointset version of the Lusin theorem that would be acceptable in Brouwer's Intuitionism. As a corollary we also obtain a version of Wim Veldman's result concerning an Intuitionistic formulation of the Lusin theorem for positively disjoint strictly analytic sets. Here the notion of positive disjointness is the natural constructive formulation of disjointness for subsets of a set with an apartness, obtained by removing the double negation from the statement that A_1, A_2 are disjoint if their intersection is empty; i.e. whenever x_1 is in A_1 and x_2 is in A_2 then x_1, x_2 are not not apart. On Baire space the infinite sequences x_1, x_2 are apart if they differ at some stage.

On the Order of Strategy Elimination Procedures in Strategic Games

Krzysztof R. Apt[1,2]

[1] School of Computing
National University of Singapore
3 Science Drive 2, Singapore 117543, Republic of Singapore
apt@comp.nus.edu.sg
[2] Centrum voor Wiskunde en Informatica (CWI)
Kruislaan 413, 1098 SJ Amsterdam, The Netherlands
apt@cwi.nl

Abstract

In the literature on strategic games two types of strategy elimination procedures were studied: the ones based on the notions of dominance and the ones based on the notions of rationalizability. The question of order independence was extensively studied for the first class of elimination procedures but not the second.

In this lecture we discuss these issues in detail. In particular, we clarify that for finite games uniform proofs of order independence for various dominance relations can be established by appealing to Newman's Lemma from 1942 and related results on the abstract reduction systems.

In the case of rationalizability three natural reduction relations on games arise. In general, they differ and only one of them is order independent. For finite games their iterations coincide and the order independence holds for three natural systems of beliefs considered in the literature.

Expressive Power of First Order Logic for Embedded Finite Models

Oleg Belegradek

Istanbul Bilgi University
Department of Mathematics
Kurtuluş Deresi Cad 47, Dolapdere, 34435 Beyoğlu Istanbul, Turkey
olegb@bilgi.edu.tr

Abstract

In this tutorial I will consider the question which queries about finite relational databases over an infinite structure are expressible in first order logic. Even though the question came from database theory, it is model-theoretic in its nature, and the answer to it depends on model-theoretic properties of this infinite structure. In the minicourse I will present known and some recent results on the problem.

A Quantified Logic of Explicit Evidence

Melvin Fitting

City University of New York
Department of Mathematics and Computer Science
Lehman College
250 Bedford Park Blvd. West Bronx, NY 10468-1589, USA
melvin.fitting@lehman.cuny.edu

Abstract

Gödel suggested the creation of a propositional logic of explicit proofs, and recently this suggestion was carried out by Artemov, to produce the logic LP. This logic can, in fact, be looked at more generally as a logic of explicit evidence. A major result about LP is the Realization Theorem, that says any theorem of S4 can be converted into a theorem of LP by some replacement of necessitation symbols with explicit proof terms. Thus the necessitation operator of S4 can be seen as a kind of implicit existential quantifier: there exists explicit evidence such that.... I introduce quantification over evidence into LP, and show that the connection between S4 necessitation and the existential quantifier is an explicit one. The extension of LP with quantifiers is called QLP. I will sketch its semantics and axiomatization.

One Hundred Years of Descriptive Set Theory

Alekos Kechris

California Institute of Technology
Department of Mathematics
Pasadena, CA 91125, USA
kechris@its.caltech.edu

Abstract

H. Lebesgue's paper "Sur les fonctions representables analytiquement", published in 1905, can be viewed as the beginning of descriptive set theory, the study of definable sets and functions in Polish spaces. In this talk, I will give an overview of the development of this subject during its first 100 years.

Phase Transitions in Satisfiability and Coloring

Lefteris M. Kirousis[1,2]

[1] University of Patras
Computer Engineering and Informatics Department
University Campus, GR-265 04 Rio, Patras, Greece
kirousis@ceid.upatras.gr
[2] RA Computer Technology Institute

Abstract

The instances of k-SAT (satisfiability of a Boolean formula in conjunctive normal form with k literals per clause), coloring and other decision problems can be partitioned according to the value of a control parameter that measures the density, with respect to the size of the problem, of the constraints that a witness which verifies the admittance of an affirmative answer to the problem should satisfy (satisfying truth assignment are examples of such witnesses). It is known that for each class of the partition, the asymptotic value of the percentage of the instances that admit an affirmative answer undergoes an abrupt change ("phase transition") from 1 to 0, as the control parameter increases. It was also experimentally observed, but a formal proof still eludes many years of research efforts by several groups, that this sharp phase transition is centered on a threshold value. Nevertheless, the value of the putative threshold has been rigorously approximated, or for the case of k-SAT or k-coloring has been computed asymptotically with respect to k. Upper bounds of such thresholds are usually estimated by computing the expected value of the density of various restricted classes of witnesses and then requiring that this expected value vanishes asymptotically (first moment method). On the contrary, originally at least, lower bounds were estimated by designing algorithms that aim at constructing witnesses and then computing the values of the density for which the algorithms succeed for almost all instances. Recently however, in a series of seminal papers, an analytic method for estimating lower bounds was introduced. The main idea of this technique is to define a suitable class of witnesses whose expected cardinality grows large, but its variation remains within limits by excluding witnesses that contribute disproportionately to their expected number (second moment method). It is only classes of witnesses having intrinsic symmetry properties that seem to satisfy the requirement of a controlled variation. Such classes however are often either not amenable to probabilistic analysis or have vanishing expected cardinality. After surveying the original results, we will give a recent application of the technique that shows that almost all 5-regular graphs are 3-colorable.

Inductive Definability and Finite-Variable Logics: From Logic to Computer Science

Phokion G. Kolaitis

IBM Almaden Research Center
Computer Science Principles and Methodologies
650 Harry Road, San Jose, California 95120-6099, USA
kolaitis@almaden.ibm.com

Abstract

The aim of this talk is to present an overview of the connections between inductive definability and finite-variable logics with emphasis on the uses of these two formalisms in finite model theory, relational databases, and computational complexity.

Dynamics of Network Worms

Evangelos Kranakis

Carleton University
School of Computer Science
Herzberg Building, 1125 Colonel By Drive, Ottawa, Ontario, K1S 5B6 Canada
kranakis@scs.carleton.ca

Abstract

Network worms are self-propagating, and self-replicating softaware that infect remote machines world-wide without (and despite) human intervention. In this talk we look at modern techniques for monitoring, detecting, and analyzing their dynamics and limiting their propagation.

Circularity and Recursion

Lawrence S. Moss

Indiana University
Cognitive Science Program
819 Eigenmann, 1910 E. 10th St., Bloomington, IN 47406-7512 USA
lmoss@indiana.edu

Abstract

My talk will be about the interaction and mutual influence of two areas: the general approach to recursion that has been one of the fields where Yiannis Moschovakis has left a permanent influence, and also the study of coalgebra that began with Peter Aczel's work on non-wellfounded sets and subsequently became more sophisticated with the development of coalgebra. Specific topics include (a) coalgebraic interpretations of FLR_0; (b) a sequent system for reasoning about sets (joint work with Glen Whitney, like me, a former student of Yiannis Moschovakis); and (c) a new general semantics of recursive program schemes, one that could be suggestive for future developments connected to FLR (joint work with Stefan Milius).

Fragments of Language, Fragments of Logic

Ian Pratt-Hartmann

University of Manchester
School of Computer Science
Manchester M13 9PL, United Kingdom
ipratt@cs.man.ac.uk

Abstract

By a "fragment" of a natural language, we understand a subset of the grammatical sentences of that language, equipped with semantics specifying the conditions under which those sentences are true. Thus understood, the familiar concepts of satisfiability and entailment can be defined for any such fragment in the expected way. The question then arises, for a given fragment of some natural language, how to determine satisfiability and entailment within that fragment.

This is, of course, an old question: Aristotle's Prior Analytics proposes, in effect, a solution to the entailment problem for very restricted fragments of some natural languages based on a finite collection of sentence schemata. However, modern developments in mathematical logic, natural language semantics and computational complexity theory afford us a perspective unavailable to the ancients. Three features characterize this new perspective. The first is generality: we view the language of the syllogism as just one of many fragments for which the satisfiability and entailment problems can be posed. The second is the requirement of validation: we demand that any method of determining satisfiability in some fragment of a natural language be accompanied by a formal assurance of its correctness. The third is an interest in computation: one of the key issues regarding any fragment of natural language is the complexity class of the satisfiability and entailment problems for that fragment.

The purpose of this short course is to describe some of the technical results that these modern developments have made available. Specifically, we present a series of fragments of English for which the satisfiability problem is polynomial, NP-complete, EXPTIME-complete, NEXPTIME-complete and undecidable. Thus, this course represents a case study in how to approach the problem of determining the logical complexity of various natural language constructions. In addition, we draw some general conclusions about the relationship between natural language and formal logic.

Direct Strictification of Categories

Zoran Petrić

Serbian Academy of Sciences and Arts
Mathematical Institute
Kneza Mihaila 35, P.O. Box 367, 11001 Belgrade, Serbia and Montenegro
zpetric@mi.sanu.ac.yu

Abstract

It is usual in mathematics to identify, tacitly, some isomorphic but different objects. For example, the two associations of cartesian product of sets $X\times(Y\times Z)$ and $(X\times Y)\times Z$ need rarely to be distinguished. In this identification, one usually identifies the canonical isomorphism between identified objects with the identity arrow. In category theory, this process should be called *strictification.* More precisely, a monoidal category is *strict* if for all of its objects the equations $A\otimes(B\otimes C) = (A\otimes B)\otimes C$ and $A\otimes I = I\otimes A = A$ hold, where I is the unit object, and moreover, the canonical arrows of the types $A\otimes(B\otimes C) \to (A\otimes B)\otimes C$, $A\otimes I \to A$ and $I\otimes A \to A$ are identity arrows. For every monoidal category, there is a strict monoidal category and an equivalence between them that preserves the monoidal structure up to some point (Joyal, Street, Mac Lane). This strictification is not an internal identification of canonical isomorphisms with identities, but it is a kind of enlarging of the existing monoidal structure into a strict one.

Usually, the goal of strictification is to obtain a category in which it is easier to record computations. There are several examples in the literature where a different kind of strictification, which is more direct and does not follow the above procedure, has been used for the same reasons. On the other hand, there is a warning example due to Isbell showing how dangerous it may be to make such a direct strictification. This talk is about conditions guaranteeing the safeness of direct strictifications of categories. It will also show that strictification is not a peculiar property of monoidal categories, and it will point to a connection with general proof theory.

Randomized First Order Computability

Dimiter Skordev

Sofia University
Faculty of Mathematics and Informatics
Sofia 1164, Bulgaria
skordev@fmi.uni-sofia.bg
http://www.fmi.uni-sofia.bg/fmi/logic/skordev/

Abstract

In his path-breaking paper [1] Yiannis Moschovakis introduced and studied two important notions of first order computability for partial multiple-valued functions on an arbitrary set, namely the notions of prime and search computability. The unprejudiced way of operating with multiple-valued functions in that paper stimulated the present author to study a generalization of some part of computability theory to other function-like objects. The generalization was announced in the paper [2], and a detailed presentation was given in the book [3] (as well as in an earlier book in Russian). Both computability notions studied in [1] turned out to be instances of the considered general one, hence the results of the above-mentioned generalization of computability theory apply to them.

A lot of other examples were indicated, and some of them are of a probabilistic nature. One such probabilistic example was examined more thoroughly, namely a case of randomized computability in the set $\mathbb{N}$ of the natural numbers (cf. Theorem 1 on pp. 251–252 of [3]). The function-like objects considered in that particular case are non-negative real-valued function θ such that $\mathrm{dom}(\theta) = \mathbb{N}^2$ and for any x in $\mathbb{N}$ the inequality

$$\sum_{y=0}^{\infty} \theta(x, y) \ \leq \ 1$$

holds. Any such θ is regarded as a representation of a randomized partial function in $\mathbb{N}$ such that for any x and y in $\mathbb{N}$ the number $\theta(x, y)$ is the probability of returning the value y when x is given as argument, and the above sum is the probability of returning some value at all for the given x. The random number generator used in the computations is supposed to produce 0's and 1's with equal probabilities, and a characterization of the computable objects θ is given in terms of recursive enumerability. Intuitively, these objects represent the randomized partial functions in $\mathbb{N}$ that are computable by means of serial computational procedures allowed to use the random number generator in question.

Considerations of a similar kind (assuming the use of the same random number generator) will be presented in the talk, but about the case of non-negative

real-valued functions θ such that $\mathrm{dom}(\theta) = (B^*)^2$ and for any x in B^* the inequality

$$\sum_{y \in B^*} \theta(x, y) \ \leq\ 1$$

holds, where B is an arbitrary set, and B^* is constructed as in [1]. The objects θ that are computable in some given partial functions in B^* will be studied. Intuitively, these objects represent the randomized partial functions in B^* which are computable by means of serial computational procedures allowed to use the given partial functions and the considered random number generator. No straightforward analog of the result concerning randomized computability in $\mathbb{N}$ holds for them without some additional assumption. However, they will be characterized through prime computability of the function value on the base of the value of the argument and certain finite sequences of 0's and 1's, the probability of returning any concrete function value depending on the set of all such finite sequences which lead to that value for the given value of the argument. Of course, since the computability investigated in this example is a particular instance of the general notion from [2, 3], the corresponding generalized results of computability theory are applicable also to that kind of computability.

References

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