Compilation to Quantum Circuits for a Language with Quantum Data and Control

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Outline

Quantum programming languages (i)

- Quantum algorithms: Shor's factoring algorithm, Grover's algorithm for database search, etc.
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- Unlike classical algorithms, quantum algorithms are usually studied at a low-level: quantum circuits or their direct mathematical abstractions
- New high-level programming languages are needed
  - They should allow programmers to use the new power of the quantum computational model
  - They should respect the special restrictions of this model
  - but they should not expose the intricacies of the model to the programmers

Quantum programming languages (ii)

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- Many important problems remain, concerning the quantum computational model and its implementation, e.g.
  - quantum error correction
  - approximation of transformations in circuts using a finite set of quantum gates
- Similar problems about the classical programming model have been solved
- Such problems should not surface in the context of (high-level) programming languages

Quantum programming languages (iii)

A brief (and certainly incomplete) summary...

- Quantum pseudocode: Knill, 1996
- qGCL: Sanders and Zuliani, 2000
- QCL: Ömer, 2003
- λ-calculus for quantum computation: van Tonder, 2004

- "Quantum data and classical control"
  - QPL: Selinger, 2004
  - λ-calculus extending QPL: Selinger and Valiron, 2005
  - Quipper: Green *et al.*, 2013

- "Quantum data and control"
  - QML: Altenkirch and Grattage, 2005, 2011
  - QIO monad in Haskell: Altenkirch and Green, 2009
  - Arrow calculus for quantum programming: Vizzoto *et al.*, 2009

Quantum computing (i)

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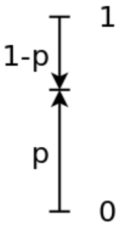

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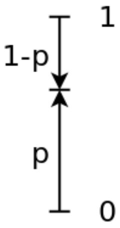

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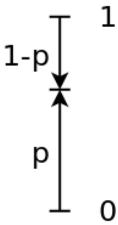

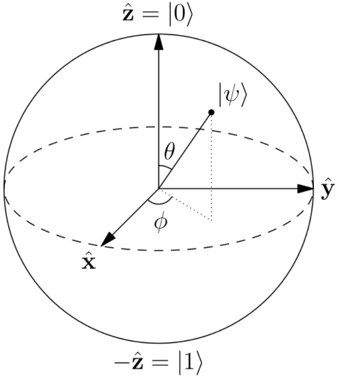

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- Entanglement
  - e.g. $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

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- Reversibility!

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6. Apply *had* to i: it yields $|0\rangle$ iff $f(0) = f(1)$!

nQML — Yet another QPL? (i)

- nQML: a quantum programming language with "quantum data and control" (Lampis *et al.*, 2006, 2008)
- Based on QML (Altenkirch and Grattage, 2005)
- Its design goals:
  - to give programmers sufficient expressive power to implement quantum algorithms easily
  - while preventing them from breaking the rules of quantum computation

nQML — Yet another QPL? (ii)

- Simple type system and denotational semantics
  - Both use structures and techniques typical in the study of classical programming languages
  - The type system does not use linear types
  - The denotational semantics uses density matrices to describe quantum states

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- Straightforward implementation in Haskell
  http://www.softlab.ntua.gr/~nickie/Research/nqml/

Overview of nQML (i)

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- Binding construct: $\mathbf{let}\ x = e_1\ \mathbf{in}\ e_2$
- Products: (e_1, e_2) $\mathbf{let}\ (x_1, x_2) = e_1\ \mathbf{in}\ e_2$

Overview of nQML (ii)

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e.g.

$$\begin{aligned} not(q) &\equiv |q\rangle \rightarrow x,y.\ \textbf{if}\ y = x\ \textbf{then}\ 0\ \textbf{else}\ 1 \\ had(q) &\equiv |q\rangle \rightarrow x,y.\ \textbf{if}\ x\ \textbf{then}\ (\textbf{if}\ y\ \textbf{then}\ -\tfrac{1}{\sqrt{2}}\ \textbf{else}\ \tfrac{1}{\sqrt{2}})\ \textbf{else}\ \tfrac{1}{\sqrt{2}} \end{aligned}$$

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add(r,c) &\equiv |r\rangle \rightarrow x,y.\ \textbf{if int } y = \textbf{int } x + c \textbf{ then } 1 \textbf{ else } 0
\end{aligned}$$

Type system of nQML

- A type τ keeps track of the exact qubits of the state in which the value of an expression is stored

$$\tau \ ::= \ \mathbf{qbit}[n] \ | \ \tau_1 \otimes \tau_2$$

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- Typing relation
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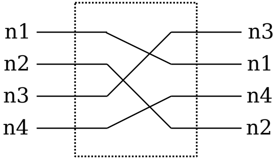
  - For impure quantum expressions $\Gamma; n \vdash e : \tau; m$
- n is the number of qubits of the original quantum state, before e starts evaluating
- m is the number of new qubits that are allocated during the evaluation of e

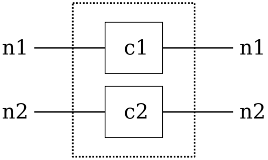
- Rotation: introduces unitary transformation where $\lambda_0^* \kappa_0 + \lambda_1 \kappa_1^* = 0$

$$\begin{pmatrix} \lambda_0 & \lambda_1 \\ \kappa_0 & \kappa_1 \end{pmatrix}$$

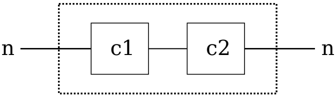
- Wire reordering



- Parallel composition

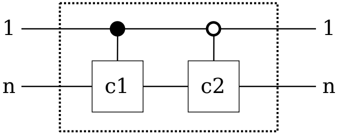


- Sequential composition

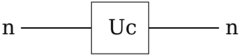


Quantum circuits (iii)

- Conditional



- Arbitrary unitary matrix



Quantum circuits (iv)

Three categories of circuits: $\mathsf{FQC}^{\approx} \subset \mathsf{FQC}^{\circ} \subset \mathsf{FQC}$
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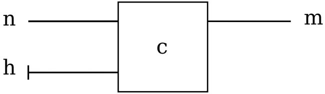

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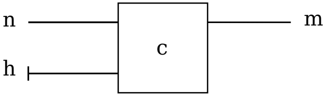
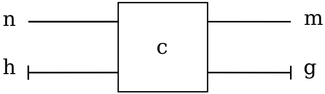
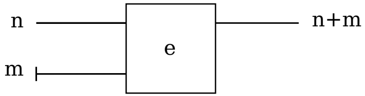


- FQC : circuits with a heap and garbage $n+h=m+g$



- Pure expressions compile to FQC$^\circ$ $\Gamma; n \vdash^\circ e : \tau; m$



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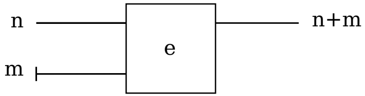
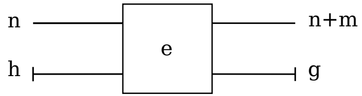
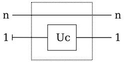


- Impure expressions compile to FQC $\Gamma; n \vdash e : \tau; m,$

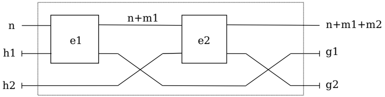


where $h = m + g$

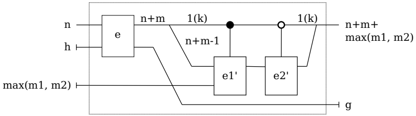
Superposition:
$\Gamma;n \vdash^{\circ} \{ (\lambda)\,\textbf{qfalse} + (\lambda')\,\textbf{qtrue} \} : \textbf{qbit}[n];1$



Let and products:
$\Gamma;n \vdash^{\alpha}$ **let** $x = e_1$ **in** $e_2 : \tau;m_1+m_2$
$\Gamma;n \vdash^{\alpha} (e_1,e_2) : \tau;m_1+m_2$
$\Gamma;n \vdash^{\alpha}$ **let** $(x_1,x_2) = e_1$ **in** $e_2 : \tau;m_1+m_2$
where:
$\Gamma_1;n \vdash^{\alpha} e_1 : \tau_1;m_1$
$\Gamma_2;n+m_1 \vdash^{\alpha} e_2 : \tau_2;m_2$



Quantum conditional:
$\Gamma;n \vdash^{\alpha}$ **if** e **then** e_1 **else** $e_2 : \tau;m+\max(m_1,m_2)$
where:
$\Gamma;n \vdash^{\alpha} e : \textbf{qbit}[k];m$
$\Gamma|_k;n+m \vdash^{\circ} e_1 : \tau;m_1$
$\Gamma|_k;n+m \vdash^{\circ} e_2 : \tau;m_2$



Measurement:
$\Gamma;n \vdash$ **ifm** e **then** e_1 **else** $e_2 : \tau;m+\max(m_1,m_2)$
where:
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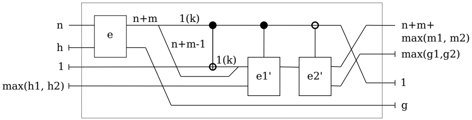
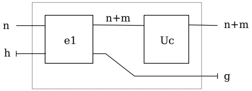


Unitary transformation:
$\Gamma;n \vdash^{\alpha} |e\rangle \rightarrow x,x'.c : \tau;m$
where:
$\Gamma;n \vdash^{\alpha} e : \tau;m$
$c(x,x')$ defines a unitary transformation on $n+m$ qubits



- Preliminaries

```
def not q = |q> -> x, x'.
  if x' = x then 0 else 1;

def had q = |q> -> x, x'.
  (if x then (if x' then -1 else 1) else 1)
    / sqrt(2);
```

- Deutsch's algorithm

```
def Deutsch f =
  let (i, j) = (had qfalse, had qtrue) in
  let r = if f i then j else not j in
  ifm had i then qtrue else qfalse;
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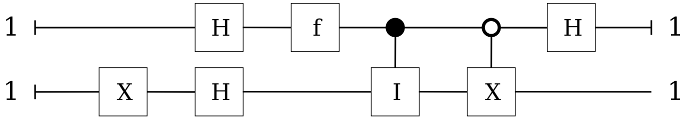


- Grover's algorithm

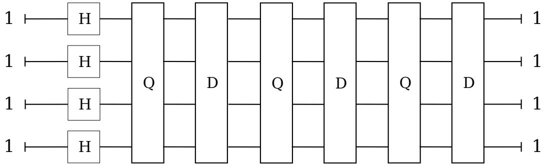
```
def query q = |q> -> x, x'.
  if x = x' then
    if int x = correct then -1 else 1
  else
    0;

def diffusion q = |q> -> x, x'.
  if x = x' then 2 / 2^n - 1 else 2 / 2^n;

def grover4 =
  let qs = (had qfalse, had qfalse,
            had qfalse, had qfalse) in
  let step1 = diffusion (query qs) in
  let step2 = diffusion (query qs) in
  let step3 = diffusion (query qs) in
  qs
```

Examples (iv)

- Grover's algorithm




```
$ ntua-qml-circ < examples/grover4
...
STATE VECTOR
 False False False False: (-5.078124999999997e-2) :+ 0.0
 False False False True : (-5.078124999999997e-2) :+ 0.0
 False False True  False: (-5.078124999999997e-2) :+ 0.0
 False False True True :  0.9804687499999996 :+ 0.0
 False True  False False: (-5.078125e-2) :+ 0.0
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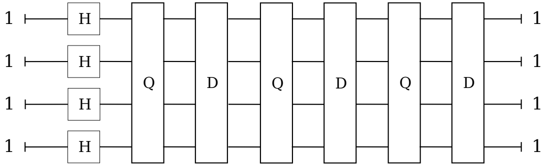


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Conclusion

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- Non linear type system; types carry qubit information
- Denotational semantics based on density matrices
- Compilation into quantum circuits in the category FQC
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Challenges for the future:

- Integration of high-level programming features
- Make quantum programming as easy as classical programming