Approximate Solution of Computationally Hard Problems: Algorithms and Complexity

Dimitris A. Fotakis

Abstract

Polynomial time approximation algorithms and complexity results for some hard optimization problems are presented in this thesis. In particular, we present approximation algorithms for special cases of the Travelling Salesman Problem (TSP), for Maximum Fault-Tolerant Partition, and Minimum Fault-Tolerant Congestion. We also obtain sufficient conditions for reducing randomness in probabilistic systems, and we prove a connection between the efficiency of these techniques and the deterministic time complexity of $\mathcal{NP}$.

We start with studying two special cases of metric TSP, Graph TSP and TSP(1,2). Initially, we show that dense instances of TSP(1,2) and Graph TSP are as hard to approximate as general instances of TSP(1,2). Then, we present an $\mathcal{NC}$ algorithm for TSP(1,2) that, for all graphs $G(V,E)$, produces a tour of length at most $|V|$ plus the cardinality of the Maximum Independent Set of G. This is the first 3/2-approximation $\mathcal{NC}$ algorithm for TSP(1,2). Next, we prove that, given a partition of a graph into constant number of cliques, optimal Graph TSP and TSP(1,2) tours can be found in polynomial time. We also obtain a $\frac{22}{15}$-approximation algorithm for TSP in graphs of diameter 3.

The algorithms for TSP(1,2) have many interesting applications to the graph radio labelling problem, that arises in the context of Frequency Assignment and is equivalent to TSP(1,2) in the complementary graph. We prove that radio labelling is in $\mathcal{P}$ for planar graphs, and approximable within 3/2 in $\mathcal{NC}$. We also provide algorithms and lower bounds for on-line radio labelling.

The Maximum Fault-Tolerant Partition problem is, given a set of channels, each having a capacity and failing randomly and independently, and a fault-tolerance constraint $(1-\epsilon)$, to compute a collection of disjoint channel subsets such that, with probability no less than $(1-\epsilon)$, at least one channel is active in each subset. The objective is to maximize the sum of the minimum subset capacities. We present a 2-approximation algorithm for identical capacities, and a $(8+o(1))$-approximation algorithm for arbitrary capacities.

In the Minimum Fault-Tolerant Congestion problem, we are given a set M of parallel channels failing randomly and independently, a set J of messages to be transmitted over M, and a fault-tolerance constraint $(1-\epsilon)$. We seek to compute a redundant assignment $\phi : J \mapsto 2^M$, that minimizes congestion, i.e. the maximum channel load, subject to the constraint that, with probability no less than $(1-\epsilon)$, all the messages have a copy on at least one active channel. Based on the results for Fault-Tolerant Partition, we present a polynomial-time 4-approximation algorithm for identical capacity channels and arbitrary message sizes, and a $2\left\lceil \frac{\ln(|M|/\epsilon)}{\ln(1/f_{\max})} \right\rceil$-approximation algorithm for related capacity channels and unit size messages.

Finally, we show that random walks on expander graphs of subexponential (in the random bit complexity) size avoid particular sets of "misleading" strings, while reducing the random bit complexity of the algorithm. Then, we introduce the idea of conditional hitting sets in order to avoid the remaining "misleading" strings. Based on the notion of conditional hitting sets, we obtain general sufficient conditions for reducing the random bit complexity of any one-side error probabilistic system. An application to $\mathcal{PCP}$ systems for $\mathcal{NP}$ suggests that, if these techniques can substantially reduce the random bit complexity of arbitrary probabilistic systems, then $\mathcal{NP}$ has subexponential deterministic simulations.

Supervisor: Professor Paul G. Spirakis

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