Approximate Mechanism Design without Money

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Social Choice and Voting

Social Choice Theory

- Mathematical **theory** dealing with **aggregation** of preferences.
- Founded by Condorcet, Borda (1700's) and Dodgson (1800's).
- Axiomatic framework and impossibility result by Arrow (1951).

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- Founded by Condorcet, Borda (1700's) and Dodgson (1800's).
- Axiomatic framework and impossibility result by Arrow (1951).
- Collective decision making, by **voting**, over anything:
  - Political representatives, award nominees, contest winners, allocation of tasks/resources, joint plans, meetings, food, ...
  - Web-page ranking, preferences in multiagent systems.

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An Example

Colors of the Local Football Club?

Preferences of the founders about the **colors** of the local club:

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Probably it would have been Red$(13) \succ$ Green$(12) \succ$ Pink(0)

A Class of Voting Rules

Positional Scoring Voting Rules

- Vector $(a_1, \ldots, a_m)$, $a_1 \geq \cdots \geq a_m \geq 0$, of **points** allocated to each **position** in the preference list.
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- **Condorcet criterion**: select the Condorcet winner, if exists.
  - **Plurality** satisfies the **Condorcet criterion**? **Borda count**?
- "Approximation" of the Condorcet winner:
  Dodgson (NP-hard to approximate!), **Copeland**, **MiniMax**, ...

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Desirable Properties of Social Choice Functions

- Onto : Range is A.
- Unanimous : If a is the top alternative in all $\succ_1, \ldots, \succ_n$, then
$$F(\succ_1, \ldots, \succ_n) = a$$
- Not dictatorial : For each agent i, $\exists \succ_1, \ldots, \succ_n$:
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- **Strategyproof** or **truthful** : $\forall \succ_1, \ldots, \succ_n$, $\forall$ agent i, $\forall \succ'_i$,
$$F(\succ_1, \ldots, \succ_i, \ldots, \succ_n) \succeq_i F(\succ_1, \ldots, \succ'_i, \ldots, \succ_n)$$

Impossibility Result

Gibbard-Satterthwaite Theorem (mid 70's)

Any **strategyproof** and **onto** social choice function on **more than 2** alternatives is **dictatorial** .

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- Randomization
- Monetary payments
- Voting systems **computationally hard** to manipulate.

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- Randomization
- Monetary payments
- Voting systems **computationally hard** to manipulate.
- **Restricted domain** of preferences – **Approximation**

Single Peaked Preferences and Medians

Single Peaked Preferences

- One dimensional ordering of alternatives, e.g. $A = [0, 1]$
- Each agent i has a **single peak** $x_i^* \in A$ such that for all $a, b \in A$:

$$b < a \leq x_i^* \quad \Rightarrow \quad a \succ_i b$$
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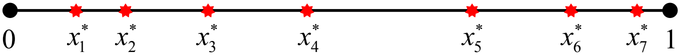

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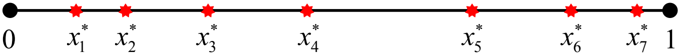
Median Voter Scheme [Moulin 80], [Sprum 91], [Barb Jackson 94]

A social choice function F on a single peaked preference domain is **strategyproof**, **onto**, and **anonymous** iff there exist $y_1, \ldots, y_{n-1} \in A$ such that for all $(x_1^*, \ldots, x_n^*)$,

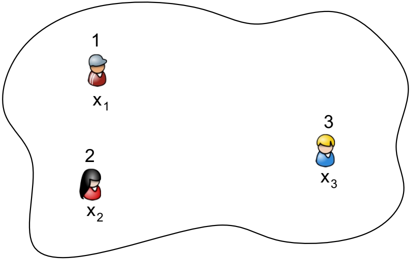
$$F(x_1^*, \ldots, x_n^*) = \text{median}(x_1^*, \ldots, x_n^*, y_1, \ldots, y_{n-1})$$



k-Facility Location Game

Strategic Agents in a Metric Space

- Set of agents $N = \{1, \dots, n\}$
- Each agent i **wants** a facility at x_i .
  Location x_i is agent i's **private information** .



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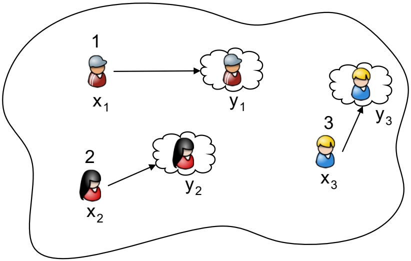
- Each agent i **reports** that she wants a facility at y_i .
  Location y_i may be **different** from x_i.



Mechanisms and Agents' Preferences

(Randomized) Mechanism

A social choice **function** F that maps a location profile $\boldsymbol{y} = (y_1, \ldots, y_n)$ to a (probability distribution over) set(s) of k **facilities** .

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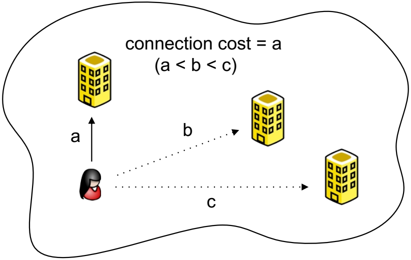
Connection Cost

(Expected) distance of agent i's **true location** to the **nearest** facility:

$$\mathrm{cost}[x_i, F(\boldsymbol{y})] = d(x_i, F(\boldsymbol{y}))$$



Desirable Properties of Mechanisms

Strategyproofness

For any location profile $\boldsymbol{x}$, agent i, and location y:

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Efficiency

$F(\boldsymbol{x})$ should optimize (or approximate) a given **objective function**.

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- Minimize *p*-**norm** of $(\text{cost}[x_1, F(\boldsymbol{x})], \ldots, \text{cost}[x_n, F(\boldsymbol{x})])$

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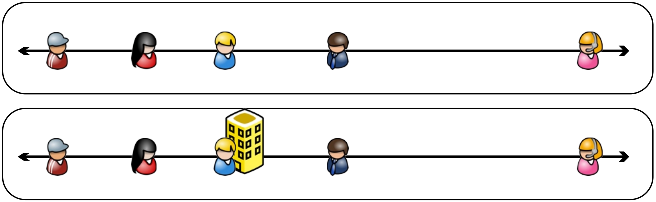
The **median** of $(x_1, \dots, x_n)$ is **strategyproof** and **optimal** .



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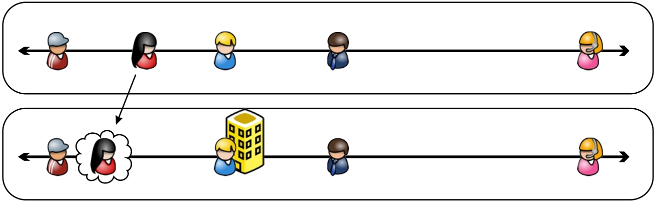

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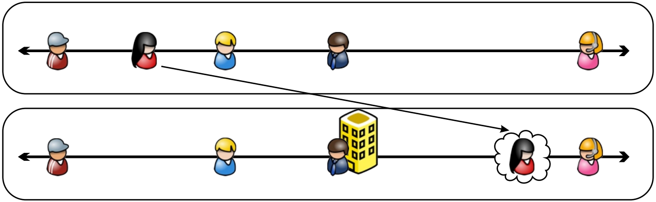

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1-Facility Location in a Tree [Schummer Vohra 02]

- **Extended medians** are the **only** strategyproof mechanisms.
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- Deterministic **dictatorship** has cost $\leq (n-1)\text{OPT}$.
- Randomized **dictatorship** has cost $\leq 2\,\text{OPT}$ [Alon FPT 10]

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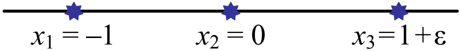

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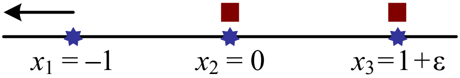

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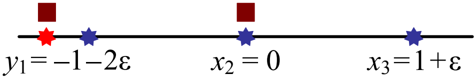

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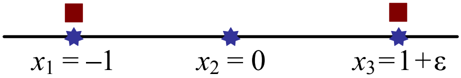
Two Extremes Mechanism [Procacc Tennen 09]

- Facilities at the **leftmost** and at the **rightmost** location:

$$F(x_1, \ldots, x_n) = (\min\{x_1, \ldots, x_n\}, \max\{x_1, \ldots, x_n\})$$

- **Strategyproof** and $(n-2)$**-approximate**.



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- Sacrifice **optimality** for **strategyproofness** .
- **Best approximation** ratio by **strategyproof** mechanisms?
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Nice mechanisms $\equiv$ deterministic **strategyproof** mechanisms with a **bounded approximation** (function of n and k).

Niceness **objective-independent** and **facilitates** the characterization!

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Dictatorial Mechanism with Dictator j

- Consider distances $d_l = x_j - \min \boldsymbol{x}$ and $d_r = \max \boldsymbol{x} - x_j$.
- Place the first facility at x_j and the second at $x_j - \max\{d_l, 2d_r\}$, if $d_l > d_r$, and at $x_j + \max\{2d_l, d_r\}$, otherwise.
- **Strategyproof** and $(n-1)$**-approximate** .

Consequences

- **Two Extremes** is the **only anonymous** nice mechanism for allocating 2 facilities to $n \geq 5$ agents on the line.
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Deterministic 2-Facility Location in General Metrics

There are **no nice** mechanisms for 2-Facility Location in metrics more general than the line and the cycle (even for 3 **agents** in a **star**).

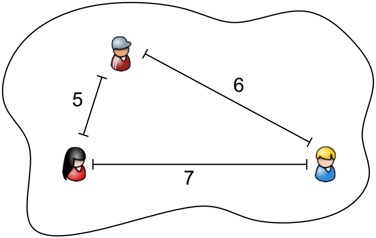
Randomized 2-Facility Location [Lu Sun Wang Zhu 10]

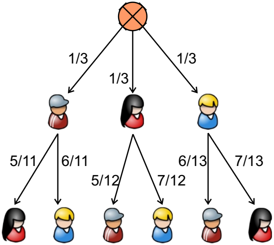
Proportional Mechanism

Facilities open at the locations of **selected agents** .

1st Round: Agent i is selected with probability $1/n$

2nd Round: Agent j is selected with probability $\frac{d(x_j,x_i)}{\sum_{\ell \in N} d(x_\ell,x_i)}$





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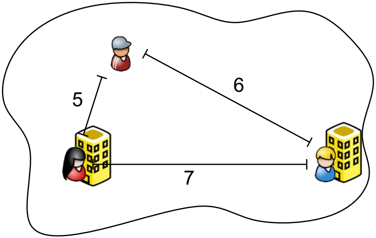

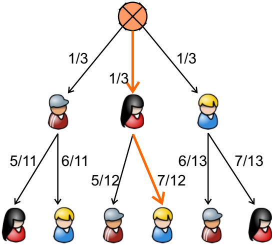

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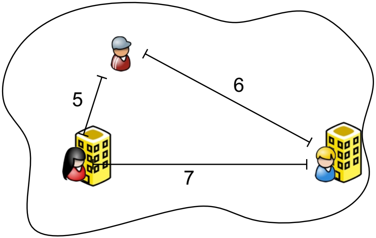

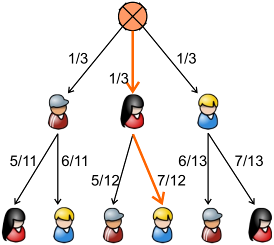

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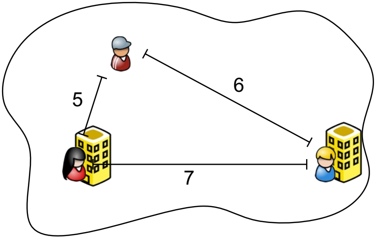
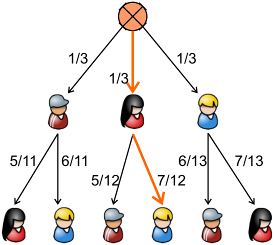
- **Strategyproof** and **4-approximate** for general metrics.
- **Not strategyproof** for > 2 facilities !
  Profile $(0 : \text{many}, 1 : 50, 1 + 10^5 : 4, 101 + 10^5 : 1)$, $1 \to 1 + 10^5$.





Randomized k-Facility Location for $k \geq 3$ [F. Tzamos 10]

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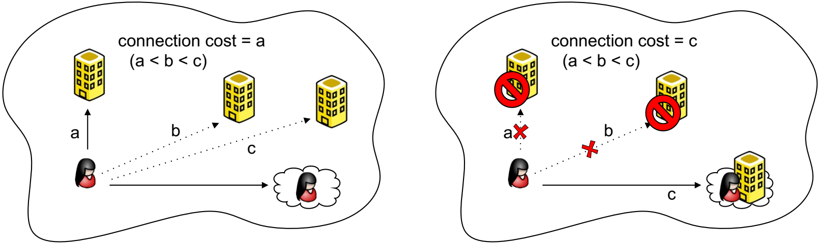
- Agents with a **facility** at their **reported** location **connect** to it. Otherwise, **no restriction** whatsoever.



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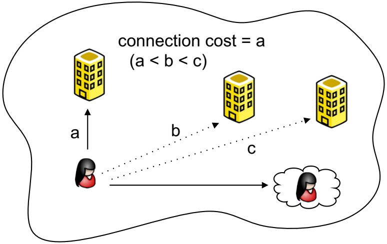
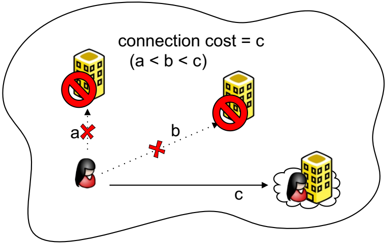
- **Winner-imposing** version of the Proportional Mechanism is **strategyproof** and **$4k$-approximate** in general metrics, for any k.





Randomized k-Facility Location on the Line [F. Tzamos 13]

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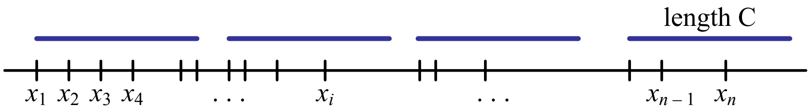
- **Optimal maximum** cost OPT $= C/2$.
- **Cover** all agents with k **disjoint intervals** of length C.



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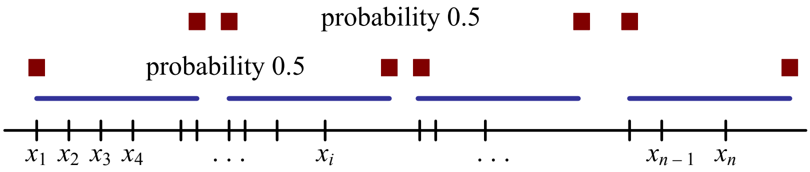

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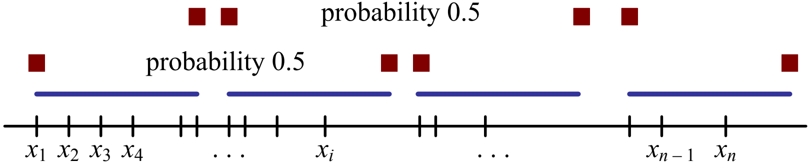

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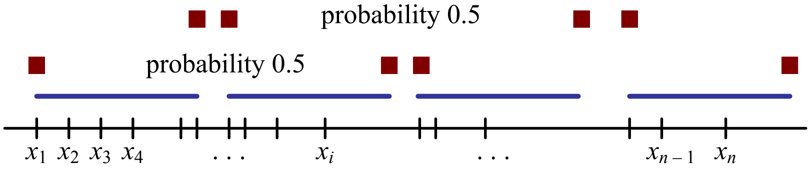
- Approx. ratio: 2 for the **maximum** cost , n for the **social** cost.



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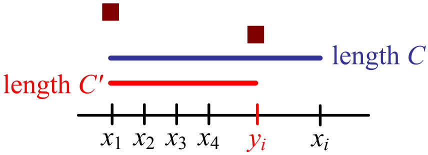

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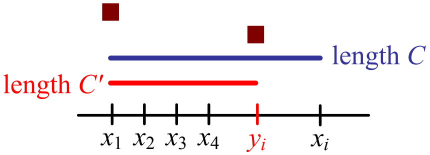

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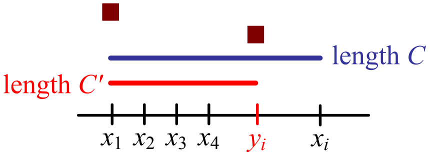

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Agents with Concave Costs

Generalized Equal-Cost Mechanism is **strategyproof** and has the **same approximation** ratio if agents' cost is a **concave function** of distance to the nearest facility.

Research Directions

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Voting and Social Networks

- How group of people **vote** for their **leader** in social networks ?
- How social network affects the people's **votes** and the outcome? Relation to **opinion dynamics** ?

Thank You!