Reasoning About Class Behavior

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Roadmap

- A Motivating Example
- Program Equivalence and Contextual Equivalence
- Technology for Proving Equivalences
- (Bisimulations)
- Deriving Bisimulations for Equivalence
- An Actual Proof of Equivalence

A Cell Class

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class Cell extends Object {
  private Object g;
  Cell() { g = null; }
  public void set(Object b) {
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  public Object get() {
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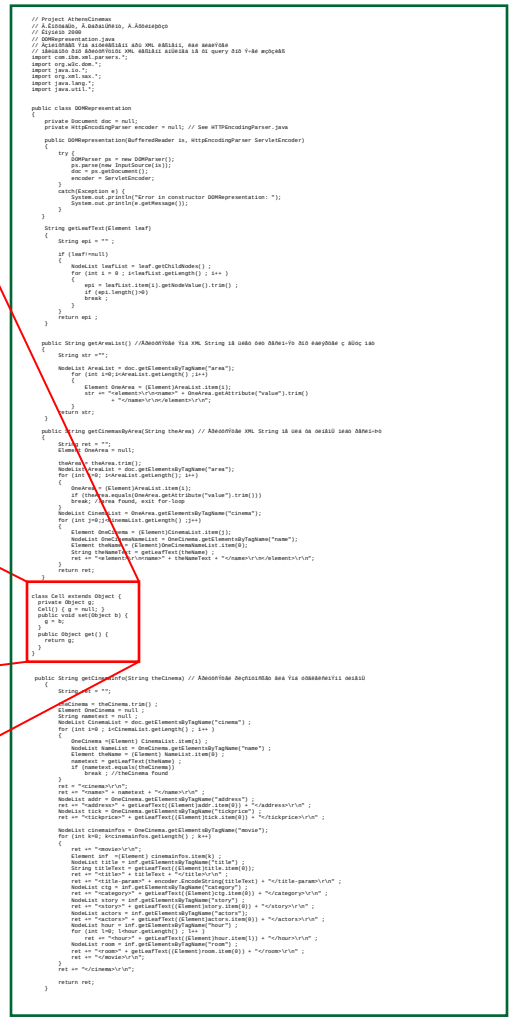

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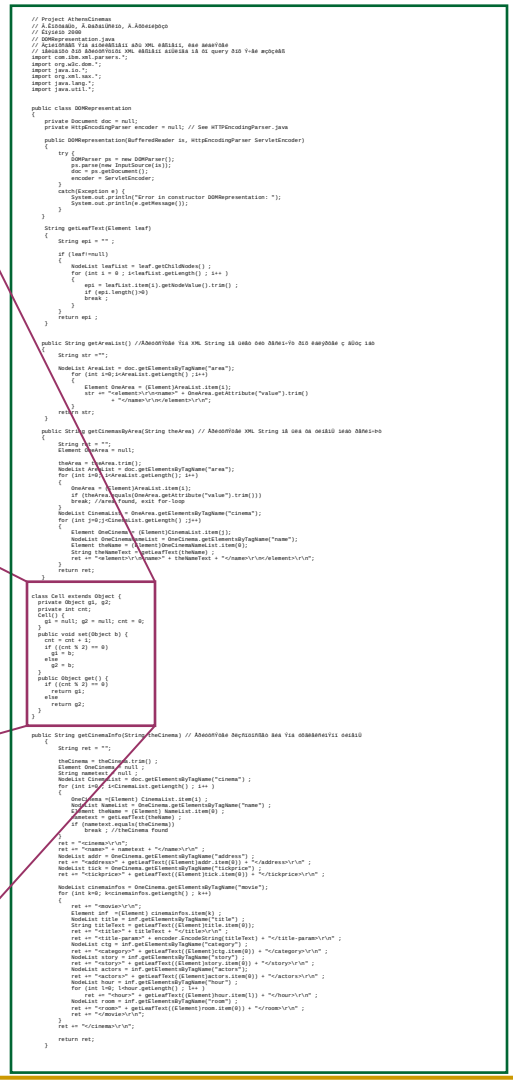

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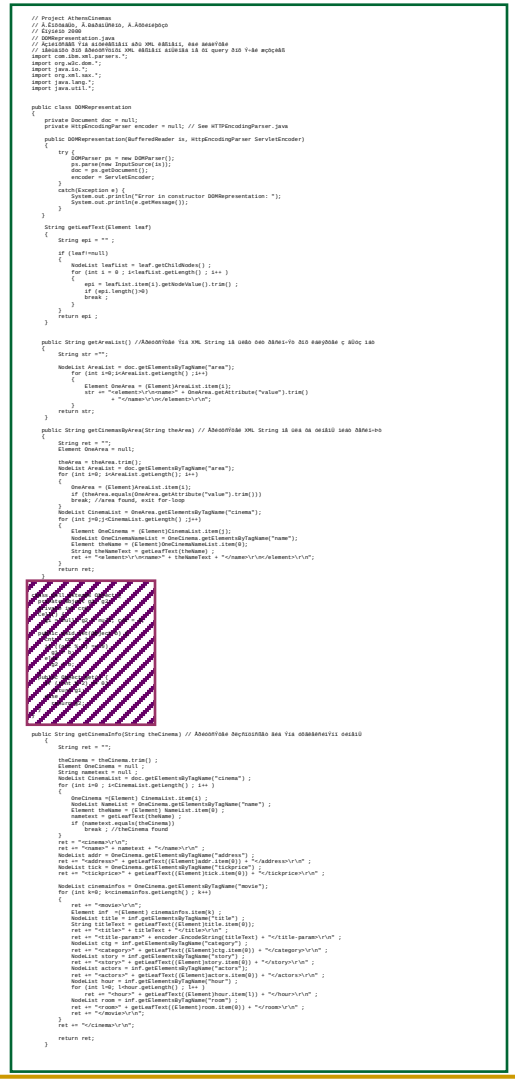
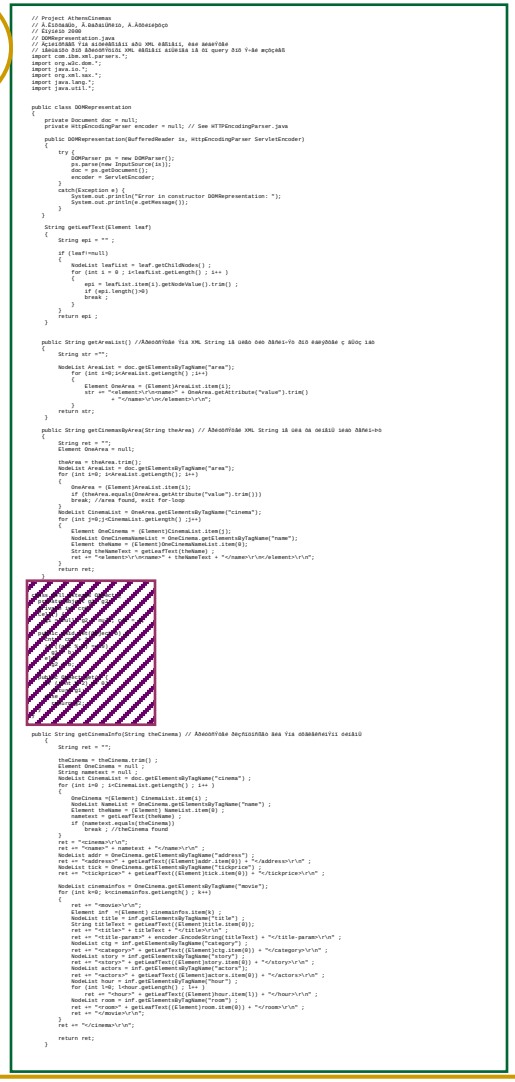


A Cell Class

Do **these two** programs have the same behavior?



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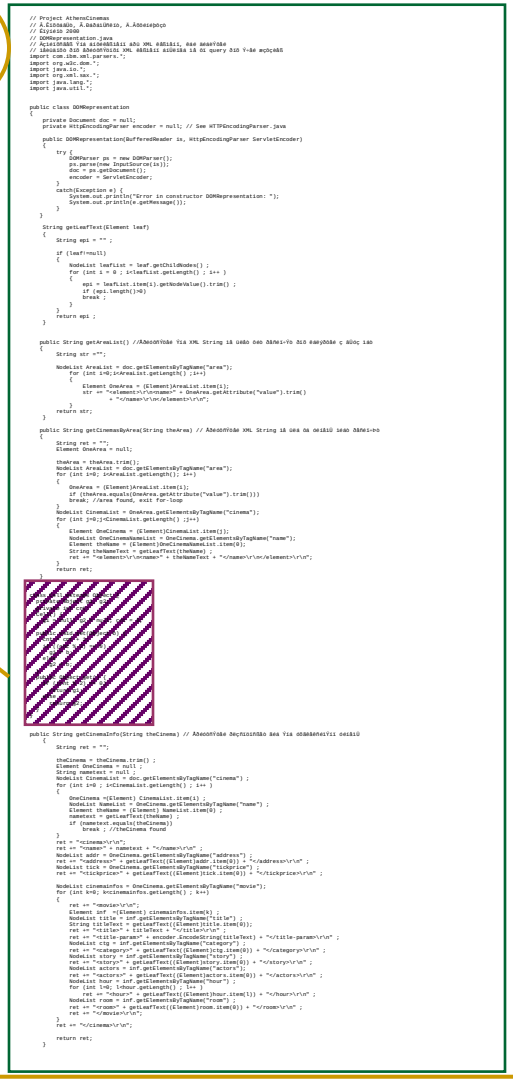
Program Equivalence



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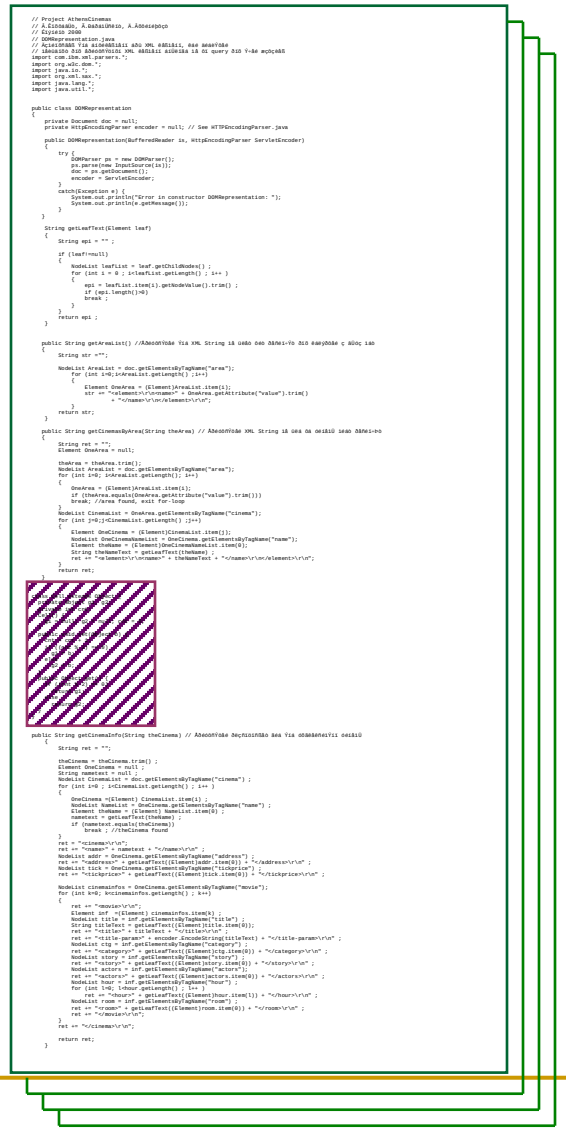

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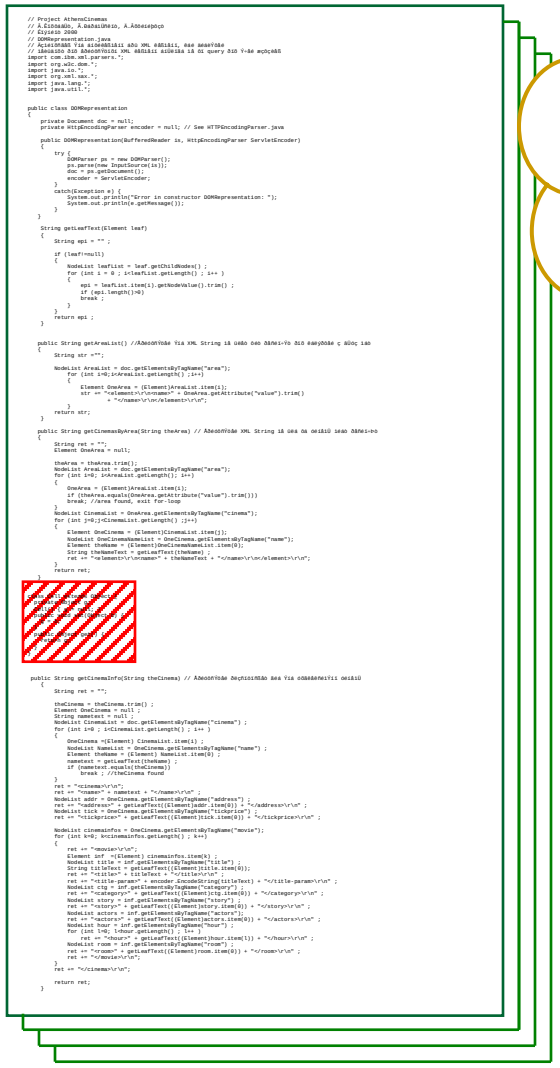

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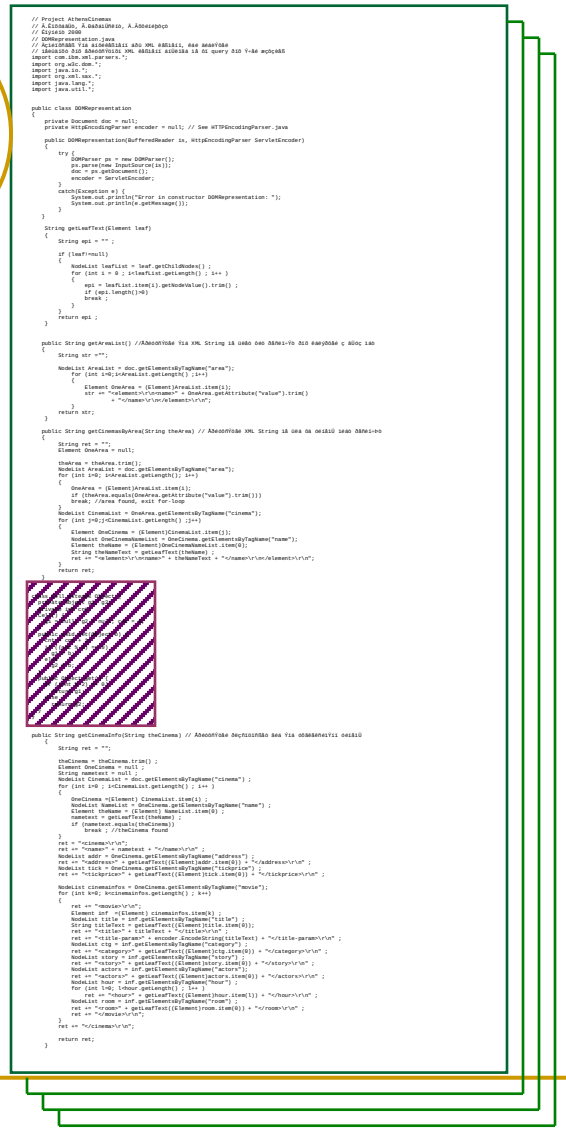

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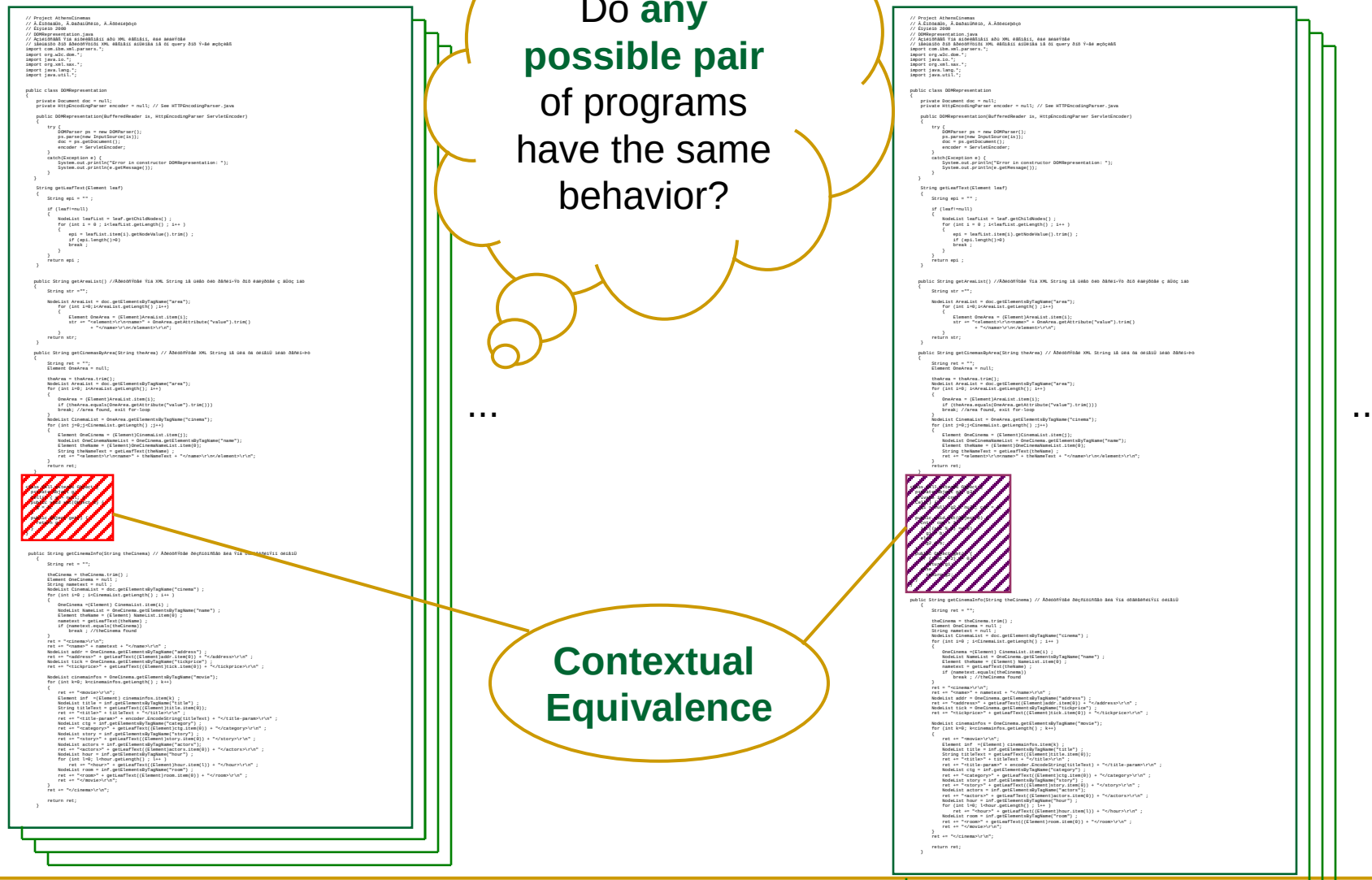
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Contextual Equivalence

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- **C** and **C'** are **contextually equivalent** iff:

For any class-table context **CT[ ]**:

CT[C] and **CT[C']** **have the same behavior**

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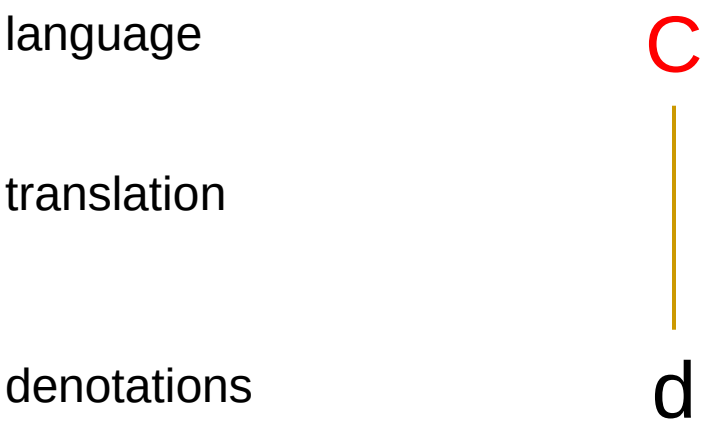
  - The "golden standard of equivalences"
  - Can't be used directly in the proof of a non-trivial equivalence
  - Can't facilitate an inductive proof

Technology for Proving Cxt Equivalence

- Denotational Semantics



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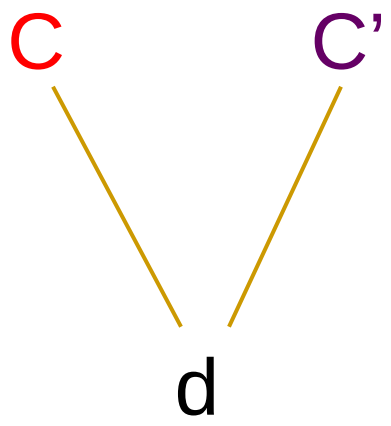

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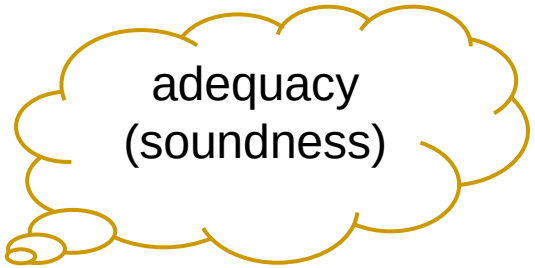

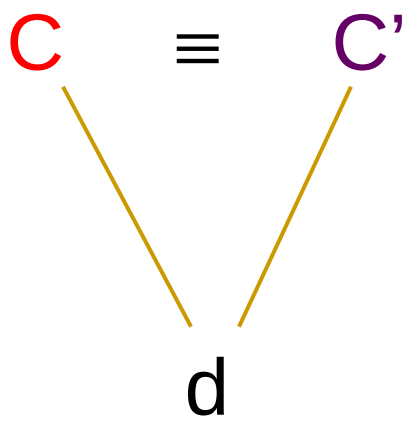

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$$C \equiv C'$$

d

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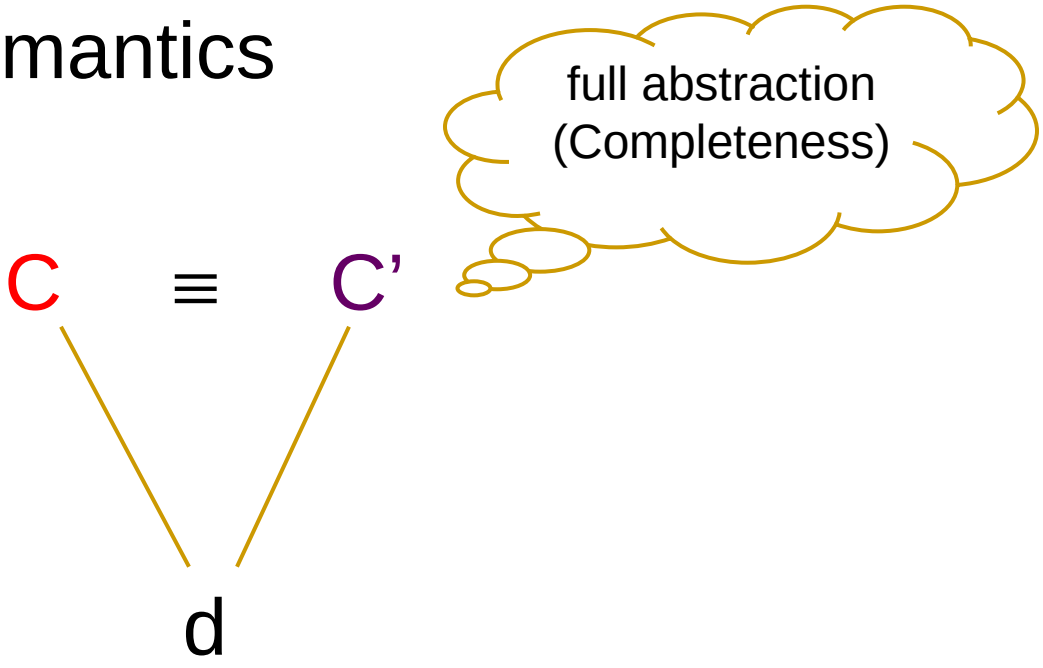

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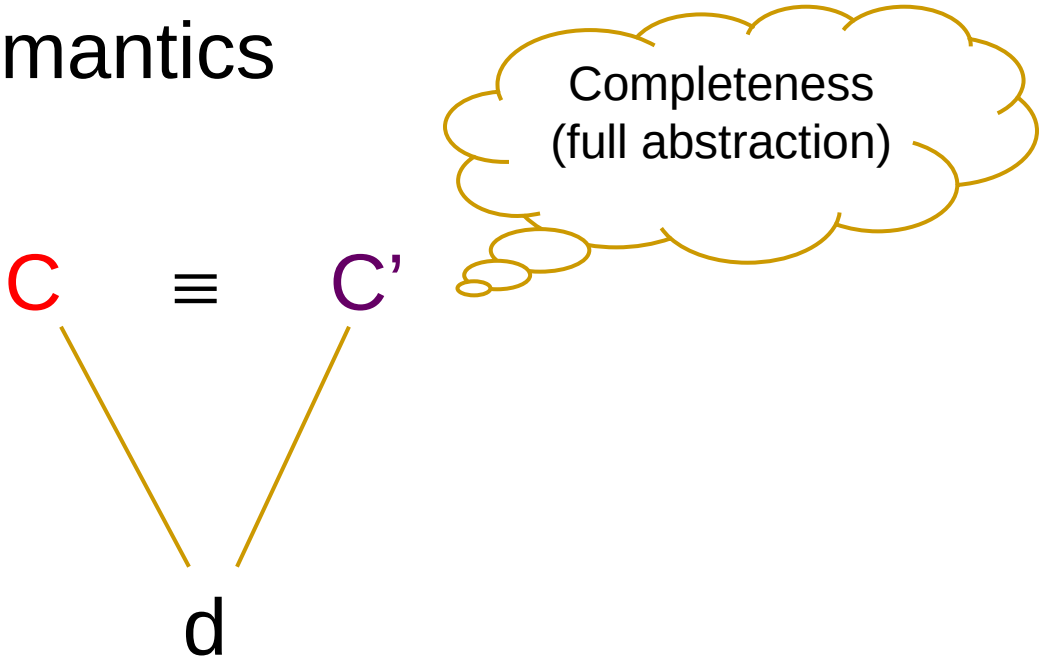
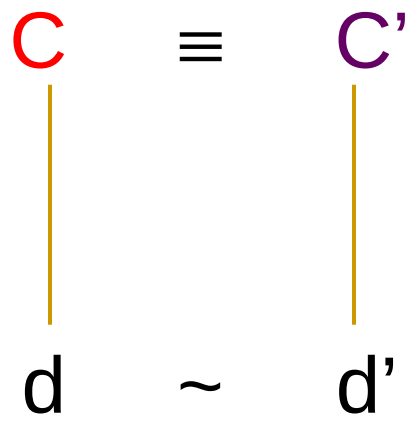


  - Usually not fully abstract when modelling languages with store

Technology for Proving Cxt Equivalence

- Denotational Semantics + Some Relation



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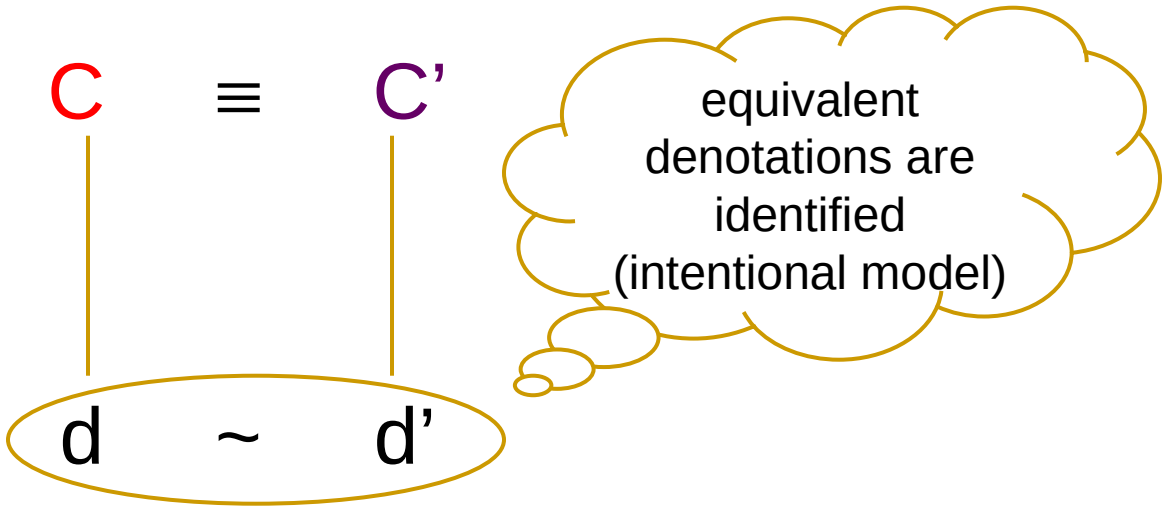

Technology for Proving Cxt Equivalence

- Denotational Semantics + Some Relation
  - Usual Denotational Semantics + Logical Relations
  - Usual Denotational Semantics + Bisimulations [Banerjee&Naumann]
  - Game Semantics + “quotient” relations [Reynolds, Hyland&Ong]

Several achieve full abstraction, but hard to use to prove some equivalences

Technology for Proving Cxt Equivalence

- Trace Equivalence [Jeffrey&Rathke]
  - Define an appropriate trace semantics (not the operational semantics of the language)

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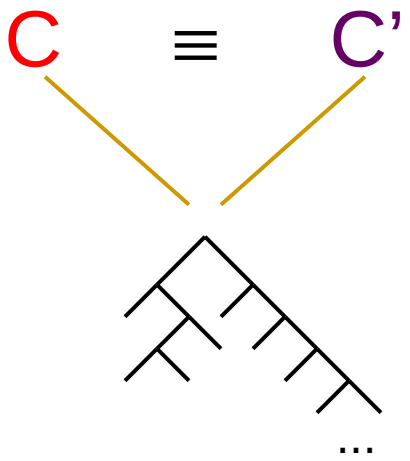

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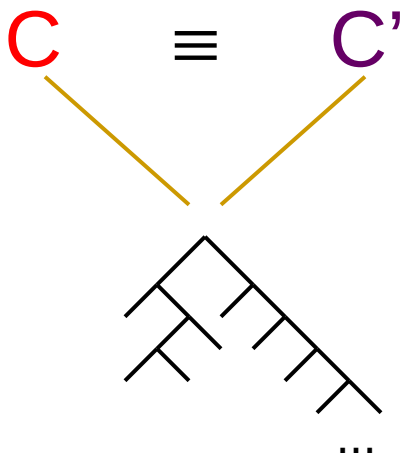


Fully abstract for Java Jr. & not so hard to use. Hard to find the appropriate trace semantics. Not obvious how to handle inheritance.

Technology for Proving Cxt Equivalence

- Operational Semantics + Logical Relations [Pitts, Pitts&Stark]
  - Fully abstract for several languages, but hard to use to prove some equivalences
- Operational Semantics + Bisimulations [Abramsky, Sumii&Pierce, Koutavas&Wand]
  - Fully abstract for diverse languages (functional, imperative)
  - Have proven all known hard equivalences
  - Not ready yet to attack complex real-world equivalences

Proving Contextual Equivalence

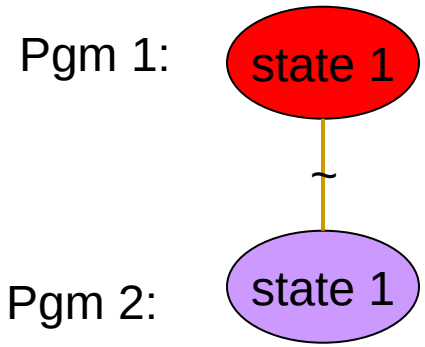
- Cxt Equivalence is the **largest set** of equivalent classes

$$(\equiv) \quad = \quad \{ (C, C') \mid \ldots \}$$

- Find a more "convenient" set R and show:
  - $R \subseteq (\equiv)$ (Soundness of R)
  - $R \supseteq (\equiv)$ (Completeness of R)

Bisimulations

- Introduced in Concurrent Calculi [Hennessy, Milner], and adapted in functional languages [Abramsky].



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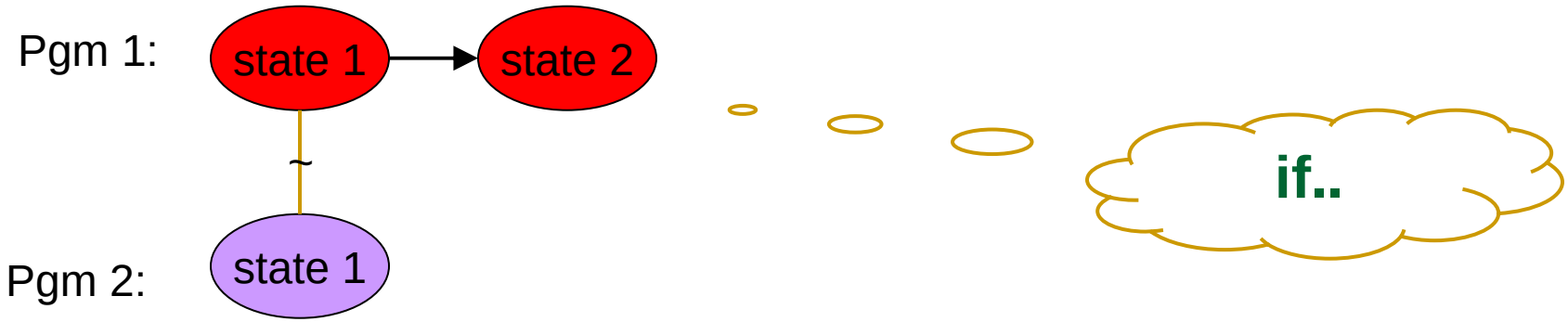

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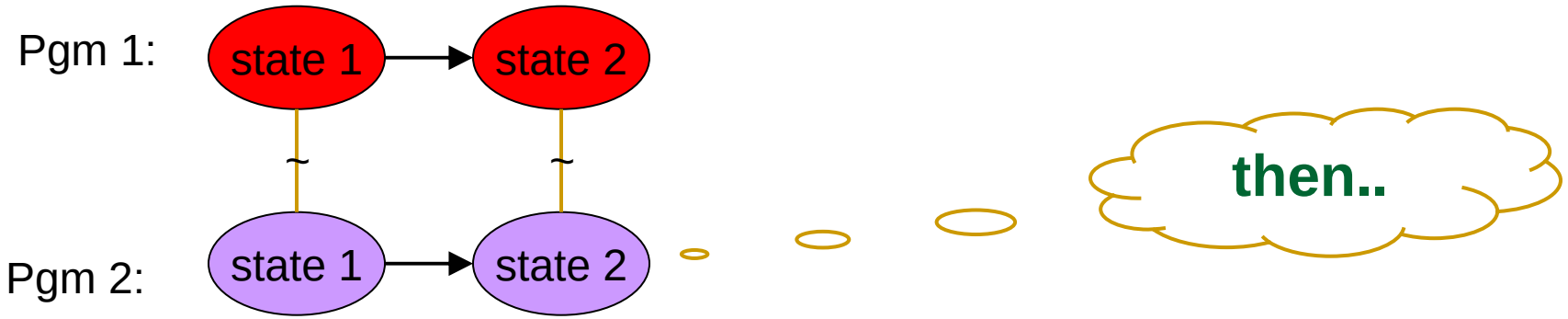

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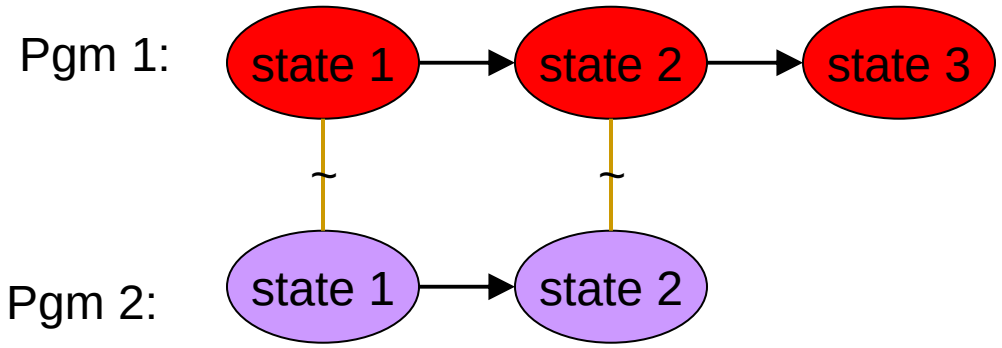

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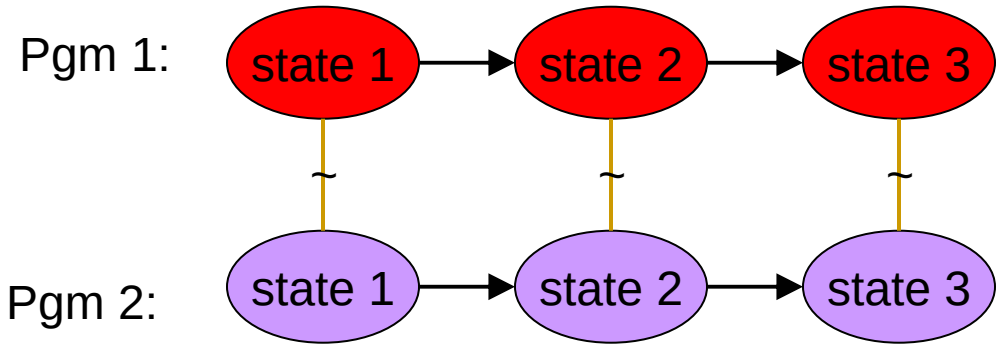

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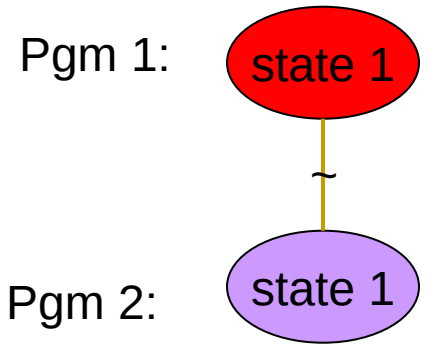

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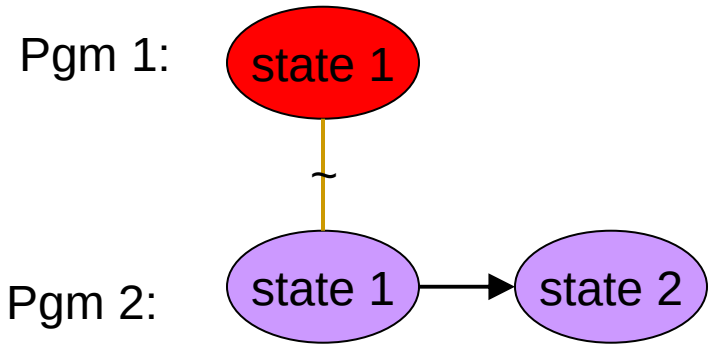

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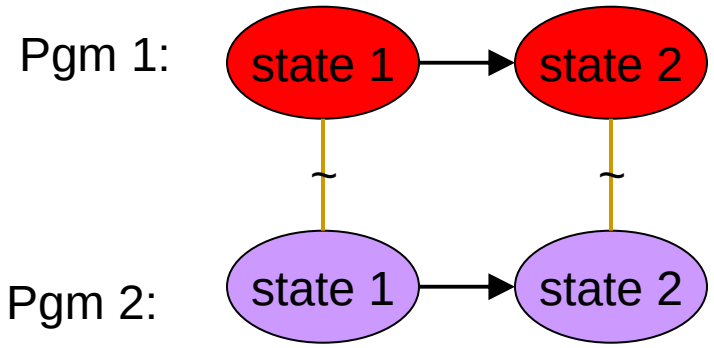

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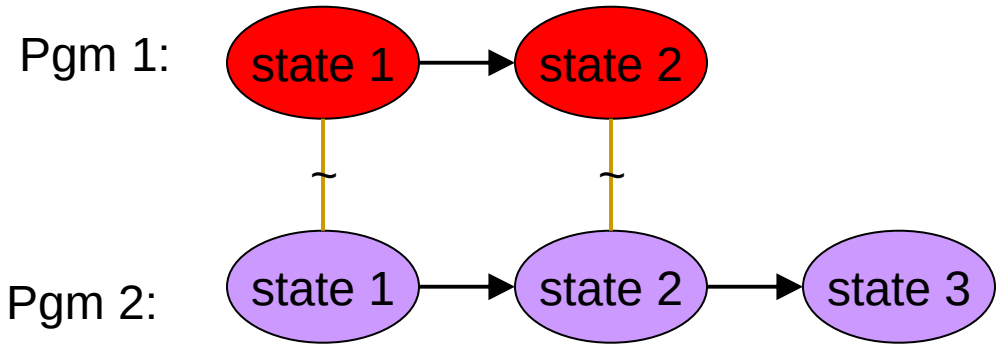

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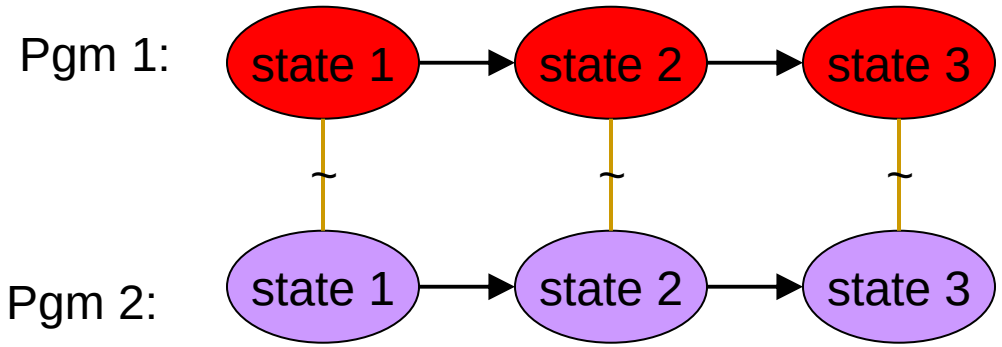

Deriving Bisimulations for Cxt Equivalence

- **C** and **C'** are **contextually equivalent** iff:

For any class-table context **CT[ ]**:

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This is a **big-step bisimulation**

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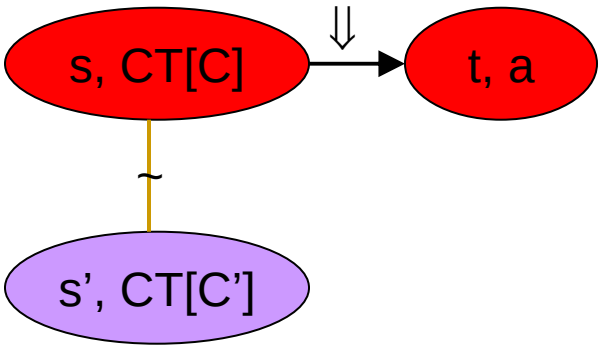

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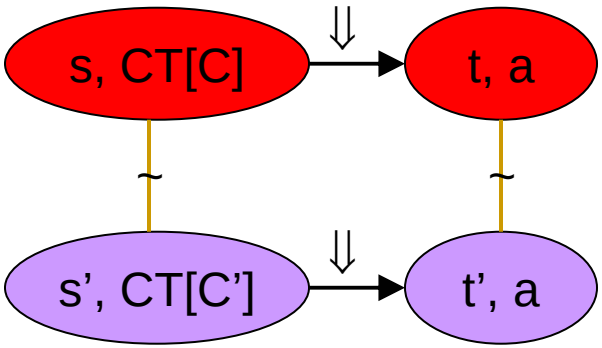

Deriving Bisimulations for Cxt Equivalence

- Thm (Soundness & Completeness):

  $(\mathbf{0}, \mathbf{0}, \mathbf{C}, \mathbf{C'}) \in (\approx)$ iff $(\mathbf{C}, \mathbf{C'}) \in (\equiv)$

- To prove C and C' equivalent we must:
  - provide a set R
  - $(\mathbf{0}, \mathbf{0}, \mathbf{C}, \mathbf{C'}) \in R$
  - $R \subseteq (\approx)$, by a **standard** inductive proof

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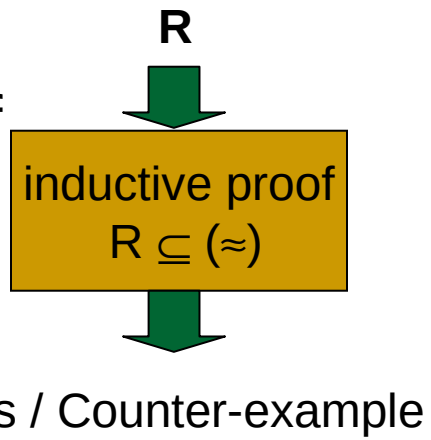

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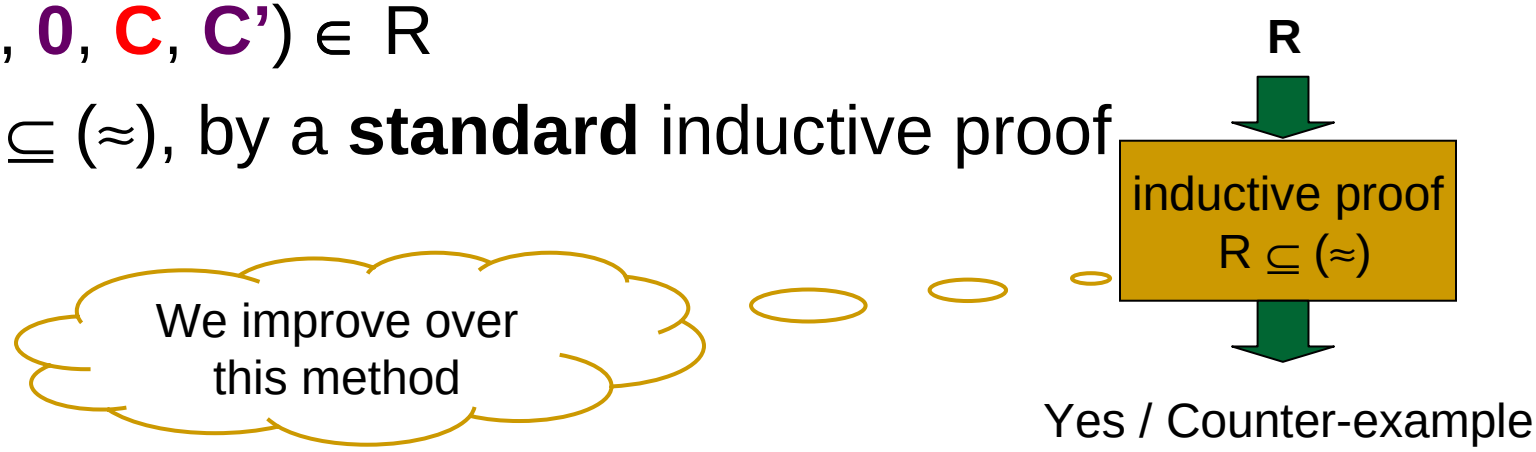
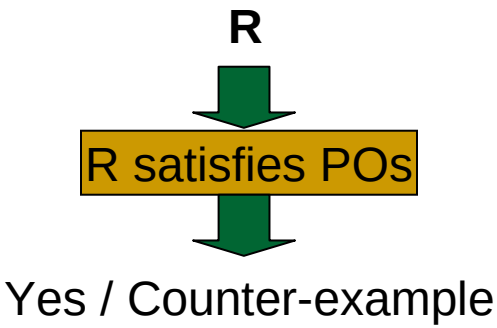


Deriving Bisimulations for Cxt Equivalence

- Inductive proof of $R \subseteq (\approx)$
  - In other words inductive proof of R being **adequate**, or R being a **bisimulation**
- For **any given R** the proof is **mostly** carried out by the induction hypothesis
  - We find **sufficient conditions** on R that can be used to **complete** the proof (**proof obligations** of R)

Deriving Bisimulations for Cxt Equivalence

- The proof of C, C' being equivalent becomes:



Proving the Equivalence of Cells

```
class Cell extends Object {
  private Object g;
  Cell() { g = null; }
  public void set(Object b)
   {
     g = b;
  }
  public Object get() {
    return g;
  }
}
```

```
class Cell extends Object {
  private Object g1, g2;
  private int cnt;
  Cell() {
    g1 = null; g2 = null; cnt = 0;
  }
  public void set(Object b) {
    cnt = cnt + 1;
    if ((cnt % 2) == 0)
      g1 = b;
    else
      g2 = b;
  }
  public Object get() {
    if ((cnt % 2) == 0)
      return g1;
    else
      return g2;
  }
}
```

Proving the Equivalence of Cells

Let R = { (s, s', Cell, Cell') |

$$s=[l= \text{obj Cell}\{g=\mathbf{l_1}\}] \bullet s_0$$
$$s'=[l= \text{obj Cell}\{g_1=\mathbf{l_1}, g_2=l_2', \text{cnt}=\mathbf{2n}\}] \bullet s_0' \ldots\}$$

PO 6: Must show that if

$$s, CT, l.get() \Downarrow t, a$$

then

$$s', CT', l.get() \Downarrow t', a$$

and (t, t', Cell, Cell') $\in$ R

Proving the Equivalence of Cells

Let R = { (s, s', Cell, Cell') |

$$s=[l= \text{obj Cell}\{g=l_1\}] \bullet s_0$$

$$s'=[l= \text{obj Cell}\{g_1=l_1,\ g_2=l_2',\ cnt=2n\}] \bullet s_0' \ ...\}$$

PO 6: Must show that if

$$s,\ CT,\ l.get() \Downarrow \mathbf{s,\ l_1}$$

then

$$s',\ CT',\ l.get() \Downarrow \mathbf{s',\ l_1}$$

and (**s, s',** Cell, Cell') $\in$ R

which is **true**.

Proving the Equivalence of Cells

Let R = { (s, s', Cell, Cell') |

$s=[l= \text{obj Cell}\{g=\mathbf{l_1}\}] \bullet s_0$

$s'=[l= \text{obj Cell}\{g_1=\mathbf{l_1},\ g_2=l_2,\ \text{cnt}=\mathbf{2n}\}] \bullet s_0'$

OR

$s'=[l= \text{obj Cell}\{g_1=l_2,\ g_2=\mathbf{l_1},\ \text{cnt}=\mathbf{2n+1}\}] \bullet s_0'$ }

Must show that for all

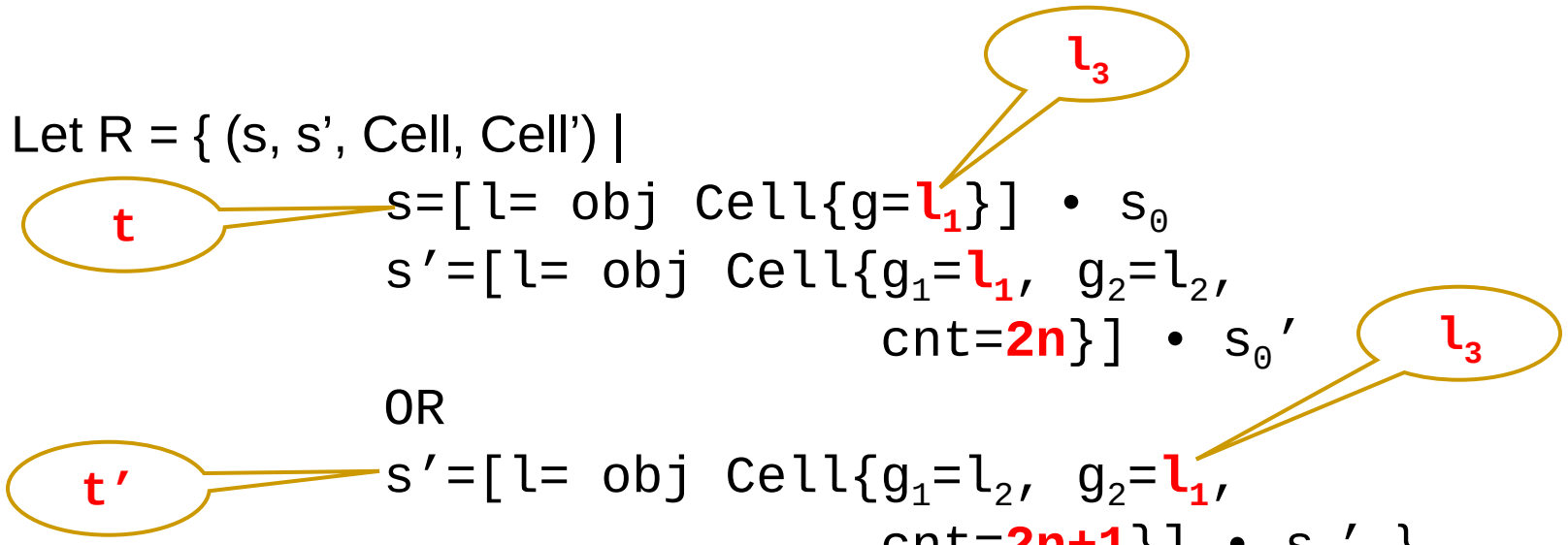
$$s,\ CT,\ l.\text{set}(l_3) \Downarrow t,\ a$$

then

$$s',\ CT',\ l.\text{get}(l_3) \Downarrow t',\ a$$

and (t, t', Cell, Cell') $\in$ R

Proving the Equivalence of Cells



Must show that for all

$$s,\ CT,\ l.set(l_3) \Downarrow t,\ a$$

then

$$s',\ CT',\ l.get(l_3) \Downarrow t',\ a$$

and (t, t', Cell, Cell') $\in$ R

Τέλος